Section 3.5: The Inverse of a Matrix

Over the set of real number we have what we call the **multiplicative inverse** or **reciprocal**. The multiplicative inverse of a number is a second number that when multiplied by the first number yields the **multiplicative identity 1**.

This is where the Identity Matrix comes in.

Let A be a square matrix of size n and another square matrix A^{-1} of size n such that $AA^{-1} = A^{-1}A = I_n$ is called the **inverse of A**.

Note: Not every square matrix has an inverse. A matrix with no inverse is called **singular**.

Finding the Inverse of a Matrix

Given the n x n matrix *A*:

1. Adjoin the n x n identity matrix *I* to obtain the augmented matrix $(A \mid I)$

2. Use the Gauss-Jordan elimination method to reduce $(A \mid I)$ to the form $(I \mid B)$, if possible.

The matrix *B* is the inverse of *A*.

Example 1: Find the inverse, if possible and check:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

Example 2: Find the inverse of a 3 x 3 matrix.(Use Gauss-Jordan)

$$C = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 3 & -2 \\ -1 & 2 & 3 \end{pmatrix}$$

Example 3: Find the inverse.

$$B = \begin{pmatrix} 4 & 2 & 2 \\ -1 & -3 & 4 \\ 3 & -1 & 6 \end{pmatrix}$$

Matrices That Have No Inverses

If there is a row to the left of the vertical line in the augmented matrix containing all zeros, then the matrix does not have an inverse. Example 3 has this problem and does not have an inverse.

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Formula for the Inverse of a 2X2 Matrix

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Suppose $D = ad - bc$ is not equal to zero. Then A^{-1} exists and is given by

$$A^{-1} = \frac{1}{D}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Example 4: Find the inverse of the following matrices.

a. $A = \begin{pmatrix} -5 & 10 \\ 2 & 7 \end{pmatrix}$

b. $B = \begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix}$

Matrix Representation

A system of linear equations may be written in a compact form with the help of matrices.

Example 5: Given the following system of equations, write it in matrix form.

$$\begin{aligned} 2x - 4y + z &= 6 \\ -3x + 6y - 5z &= -1 \\ x - 3y + 7z &= 0 \end{aligned}$$

Example 6: Write each system of equations as a matrix equation and then solve the system using the inverse of the coefficient matrix.

$$2x + 3y = 5$$
$$3x + 5y = 8$$

The use of inverses to solve systems of equations is advantageous when we are required to solve more than one system of equations **AX = B**, involving the same coefficient matrix, **A**, and different matrices of constants**, B**.

Example 7: A performance theatre has 10,000 seats. The ticket prices are either $25 or $35, depending on the location of the seat. Assume every seat can be sold.

a. How many tickets of each type should be sold to bring in a return of $275,000?
b. How many tickets of each type should be sold to bring in a return of $300,000?
Let x = number of $25 tickets and y = number of $35 tickets